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**Mathematical Modeling of Integrated Manufacturing And Distribution Systems With
Fuzzy Approach in Supply Chain Management in the Food Industry**

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Abstract

Today, the role of the economy in various spheres among food is on the rise. Fuzzy supply chain management is one of the modern attitudes that lead to reduce system costs, food spoilage, losses in shipping products and to reduce inventory levels. Management of fixed costs including the cost of distribution centers construction and variable costs such as transportation costs, maintenance, shortages, manpower, deterioration of food products and others will be concluded. Queue model structure which used in this study obeys from M/G/1 queue that arrival rate Poisson probability distribution function and the service rate have the general probability distribution function. In this study, fuzzy supply chain has been analyzed in three levels. The suppliers are in first level, the distribution centers are in second level and the local warehouse (consumer) is located in third level. The structure of objective function of the presented optimization model is based on the profit maximization which the resulted benefit from products sale obtained from the difference between total revenues and total costs. The appropriate results can be obtained from the model analysis. To determine the transfers volume of suppliers to the intermediate warehouses (distribution centers) as well as determining the volume of transfers from the central warehouse to local warehouse. The model is solved by using Maple 12 software. Finally, the proper results are stated. This approach causes the appropriate management of market, increasing of customer satisfaction and ultimately, increasing the efficiency and effectiveness of the activities in the supply chain, distribution of products in the food industry.

Keywords: Fuzzy supply chain management, Queuing theory, Mathematical modeling, Optimization, Fuzzy theory

Introduction

According to complexity of decision making at the mass level, in today's competitive world, traditional management cannot have necessary efficiency and effectiveness. Therefore, supply chain management [1] can be considered as a new management approach. The used models belong to location allocation models type. Initially, the location allocation models were presented by Cooper in 1963[2] that since then, extensive researches have been done in this area. Industries progress in the specialized field has led to confront with complex dimensions issues, uncertain in the real situation which the specific policies must be obtained in this regard. Therefore, supply chain modeling has been done by fuzzy math approach to this problem that can be used in distribution systems. Supply Chain Management is comprised of three main components that one of these components is logistics management or inventory and material flow management in the supply chain.

Logistics has different levels that each of the components plays an important role in the whole system. Tavakkoli - Moghaddam, et al. [3] studied the mathematical modeling of a location multi-objective allocation problem in three-level supply chain. Amiri [4,5] studied the designing of service delivery systems in the communication network which minimize the cost of installation, operation and expectation and used from innovative algorithms and lagrangian methods in solution. Rehman and Smith [6] used location allocation models in the development and improvement of health care centers [7] studied fuzzy Mathematical Models in location allocation models to design a three-level supply chain. Mariano [8] studied the numbers of optimized service providers in any facilitation upon the queue that it has the arrival rates of the exponential distribution function and the distribution service function that has the Erlang distribution and the limited population can entry the queue. Yazdi and Shahanaqy [9] considered the multiple targets in a mathematical model for locating distribution centers and allocation of customer demand in problems of supply chain network design. Firoozi and colleagues [10] studied a combined algorithm to develop the problems of distribution network design. [11] studied the mathematical models in distributed systems models by using the location allocation models that it is the only part of the performed research in this area. In this study, the various aspects of fuzzy mathematical models in production and distribution systems at the three-level supply chain have been studied. The structure of the objective function is based on maximizing the resulted profits of total income and the cost of transportation,

construction of distribution centers, maintenance, shortages, manpower and food products spoilage. The desired results can be obtained from the model are very necessary and useful which include of: to allocate the suppliers to the distribution centers and in the case of allocation, the optimal amount of demand that must be estimated in distribution centers and similarly, to allocate the distribution centers to the local warehouses (consumer) and in the case of allocation, the optimal amount of demand that need estimated for local storage (consumer). In the second part of the service delivery model and available relations in queuing systems are stated. In the third part of the optimization model of supply chain is presented. In the fourth section the presented mathematical model of the fuzzy model is converted to a deterministic mathematical model. In the fifth part of the optimization model has been considered. In the sixth part the model concluding are presented and in the last part the references are also provided.

Materials and Methods

Formulating service delivery model

Queuing Theory Models somehow are studied the queuing behavior in economic systems. The most obvious service systems can be indicated to the queue systems which in this system the customers get out of the system after entering the system and receiving the services. The criteria that stated in this area are Queue length, the waiting time for service, productivity rates in the queue system, the average number of people in the queue and system, the average waiting time in the queue and the system [12].

Assumptions and notation

- λ : Shipment arrival rate in a group
- μ : the service rates at each center (distribution centers and local warehouses)
- ρ : coefficient of system efficiency
- L_q : The average number of shipments in the queue
- L : The average number of shipments in the system
- W_q : The average waiting time of shipment in queue
- W : the average waiting time of shipment in system
- $E(N)$: The average number of shipments
- $E(S)$: average time for service
- N : Shipment volume

$$\rho = \lambda \cdot \frac{E(N)}{E(S)} = \lambda \cdot \frac{E(N)}{\mu}, (1) \quad W_q = \frac{L_q}{\lambda}, (2) \quad W = W_q + \frac{1}{\mu}, (3)$$

$$L_q = \frac{\rho^2 + [\lambda \cdot E(N)]^2 \text{Var}(S) + \rho \left[\frac{E(N^2)}{E(N)} - 1 \right]}{2(1-\rho)}, (4) \quad L = L_q + \rho, (5) \quad E(S) = \frac{1}{\mu}, (6)$$

Optimization Model

In this section we propound the assumptions, indices definitions, parameters, variables and the optimization model:

Assumption

- A) The cost of employing labor (service provider) in various period in distribution centers and local warehouses are the same.
- B) The demand in distribution centers which has limit is absolute (upper limit).
- C) Demand at local storage is constant and deterministic.
- D) The capacity of distribution centers and local warehouses is a fuzzy amount.

Index

- l :Manufacturers Index (l=1,2,...,L)
- j: Index of distribution centers (j=1,2,...,J)
- n: The local stock indices(consumer) and (n=1,2,...,N)
- p: Shipment index (p=1,2,...,P)
- t: Time Periods (t=1,2,...,T)

3.3.Parameters

K_{pl} : Purchase price of p type products in supplier place l

H_{pjt} : maintenance cost of each certain shipment of p type in j distribution center in t period
 H_{pnt} : maintenance cost of each certain shipment of p type in n warehouse in t period
 π_{pnt} : Shortage cost of each certain shipment of p type in n warehouse in t period
 g_j : Cost of using the manpower in the j distribution center for moving shipment and service delivery (according to M/G/1queue model)
 g_n : Cost of employing manpower in n local warehouse for moving shipment and service delivery (according to M/G/1queue model)
 Δ_j : Capacity of servicing in j Distribution Center (fuzzy numbers)
 Δ_n : capacity of servicing in n Place of warehousemen(fuzzy numbers)
 LT_{plj} : Arrival time of p type shipment from supplier l to j distribution center
 LT_{pjn} : Arrival time of p type shipment from j distribution center to n place of warehousemen
 D_{pj} : The upper limit of p type load demand at j distribution center
 D_{pn} : Shipment demand of p type in the place of n warehouse
 T_{plj} : Transportation cost of per unit of p type shipment from manufacturers l to j distribution centers at t period.
 T_{pjt} : Transportation cost of per unit of p type shipment from j distribution centers to local warehouse of consumer n at t period
 C_j : Fixed cost of j distribution center
 M_j : Number of employed service providers at j distribution center
 O_n : Number of employed service providers at location of the n warehousemen
 α : waste percentage in first stage (From suppliers to distribution centers)
 γ : waste percentage of second stage (from distribution centers to warehouses of factories)
 R_{pnt} : sale price of p type shipment at the n local store at t period
 D : Fixed number of distribution centers that they are a fixed and obvious number.

Variables

S_{lj} : if supplier l send shipment to the j distribution center. (0-1)
 F_{jn} : if j distribution center send shipment to n warehouse location. (0-1)
 W_j : if j distribution center be constructed. (0-1)
 Z_{plj} : Number of sent p type shipments from supplier l to j distribution center
 X_{pjn} : Number of sent p type shipments from j distribution center to n warehouse location
 Q_{pj} : Number of receiving p type shipments in j distribution center
 B_{pn} : Number of receiving p type shipments in n storage location

Fuzzy mathematical models

$$\begin{aligned}
 \text{Maxprofit} = & \sum_p \sum_n \sum_t R_{pnt} \cdot B_{pn} - \sum_p \sum_t \sum_j H_{pjt} Q_{pj} - \sum_p \sum_t \sum_n (H_{pnt} + \pi_{pnt}) B_{pn} \\
 & - \sum_j C_j W_j - \sum_j M_j g_j W_j - \sum_j \sum_l M_j g_j S_{lj} - \sum_j \sum_n g_n O_n F_{jn} - \sum_p \sum_l \sum_j \sum_t \\
 & (K_{pl} + LT_{plj} \cdot T_{pljt}) - \sum_p \sum_j \sum_n \sum_t (LT_{pjn} \cdot T_{pjt}) X_{pjn}, S.T.
 \end{aligned}$$

$$W_j \geq S_{lj}, (6) \quad W_j \geq F_{jn}, (7)$$

$$(1 - \alpha) \sum_p \sum_l Z_{plj} = \sum_p Q_{pj}, (8) \quad (1 - \gamma) \sum_p \sum_j X_{pjn} = \sum_p B_{pn}, (9)$$

$$\sum_j Q_{pj} \leq \tilde{\Delta}_j, (10)$$

$$\sum_j B_{pn} \leq \tilde{\Delta}_n, (11)$$

$$\sum_j W_j = D(\text{number}), (12)$$

$$\sum_p \sum_l Z_{plj} \geq \sum_j D_{pj}, (13)$$

$$\sum_p \sum_j X_{pjn} \geq \sum_n D_{pn}, (14)$$

$$S_{lj}, F_{jn}, W_j \in \{0,1\}, (15) \quad X_{pjn}, Z_{plj}, Q_{pj}, B_{pn} \geq 0, (16)$$

The first part of the objective function describes about sales results of shipments. The second part of the objective function presents about the maintenance cost of per shipment in distribution center. The third part provides the cost of maintenance and shortage of each shipment in distribution centers and local warehouses. The fourth and sixth and seventh provide the cost of service providers in local Warehouses, distribution centers. Part five provides the construction cost of distribution centers. Part eight demonstrates the purchase and transportation costs from suppliers to distribution centers. Ninth Part presents transportation costs from the distribution centers to local warehouse. Sixth and seventh limitations check the construction and non-construction of distribution centers. The Eight limitation emphasizes that the total sent unharmed shipments from suppliers to distribution centers must be equal to the number of receiving shipments. Ninth limitation highlights total sent unharmed shipments from distribution centers to warehouses of factories must be equal to the number of receiving shipments. Tenth and Eleventh restrictions are relating to capacity of each node in distribution centers and warehouses of factory. Twelve restriction says that only a limited number of distribution centers should be established. Limitations of the thirteenth and fourteenth are related to customer demand that their requirements should be answered in two and three level. Limitations of the fifteenth and sixteenth discuss about variables of the model.

To convert the fuzzy mathematical models to deterministic mathematical model

The presented model in Section 3.4, the model is provided in fuzzy form that the fuzzy mathematical models must be converted to deterministic mathematical model. So if the fuzzy mathematical model is in following form, we will have

$$MaxZ = \sum_j C_j X_j, S.t. \sum_i A_{ij} \cdot X_j \leq \tilde{B}_i, X_j \geq 0, (17)$$

According to the fuzzy model number (17), the final model is as follows. [13], [14], [15]

$$MaxZ = \beta, S.t. \beta(Z_U - Z_l) - Z \leq -Z_l, \beta \cdot P_i + \sum_j A_{ij} \cdot X_j \leq B_i + P_i, \beta \geq 0, X_j \geq 0, (18)$$

Therefore fuzzy mathematical model of the main problem converted to a deterministic mathematical model in this form (upper limit)

$$MaxZ_u = V$$

$$S.t. W_j \geq S_{lj} (6), W_j \geq F_{jn} (7), (1-\alpha) \sum_p \sum_l Z_{plj} = \sum_p Q_{pj}, (8)$$

$$(1-\gamma) \sum_p \sum_j X_{pjn} = \sum_p B_{pn}, (9) \sum_j Q_{pj} \leq \tilde{\Delta}_j, (10), \sum_p \sum_l Z_{plj} \geq \sum_j D_{pj}, (13), X_{pjn}, Z_{plj}, Q_{pj}, B_{pn} \geq 0, (16)$$

$$\sum_p \sum_l S_{lj} \cdot Z_{plj} \leq \Delta_j + R_j (19), \sum_p \sum_l F_{jn} \cdot X_{pjn} \leq \Delta_n + R_n (20)$$

$$\sum_p Q_{pj} \leq \Delta_j + R_j (21), \sum_p B_{pn} \leq \Delta_n + R_n (22)$$

Then Z_l (Lower limit of the objective function) can be calculated [1], [13], [14]

$$MinZ_l = V$$

$$S.t. W_j \geq S_{lj} (6), W_j \geq F_{jn} (7), (1-\alpha) \sum_p \sum_l Z_{plj} = \sum_p Q_{pj}, (8)$$

$$(1-\gamma) \sum_p \sum_j X_{pjn} = \sum_p B_{pn}, (9) \sum_j Q_{pj} \leq \tilde{\Delta}_j, (10), \sum_p \sum_l Z_{plj} \geq \sum_j D_{pj}, (13), X_{pjn}, Z_{plj}, Q_{pj}, B_{pn} \geq 0, (16)$$

$$\sum_p \sum_l S_{lj} \cdot Z_{plj} \leq \Delta_j (23), \sum_p \sum_l F_{jn} \cdot X_{pjn} \leq \Delta_n (24)$$

$$\sum_p Q_{pj} \leq \Delta_j (25), \sum_p B_{pn} \leq \Delta_n (26)$$

Finally, it is replaced in the model number 18 and the fuzzy mathematical model converts to a deterministic mathematical programming model.

Solving the model

Zero programming model and a mixed were solved and were analyzed by Maple 12 software. The output of suppliers designated model to the distribution centers and similarly, the allocation of distribution centers to the retailers which are located on the third level. The model problem examined as a case study which in this study considered the diversity shipments in three types and for each one of suppliers, distributors and retailer warehouse considered a suggested location that its results are described in below.

Table 1.result of the model , W1=W2=1, W3=0

| B_{pn} | n=1 | n=2 | n=3 |
|----------|-----|------|-----|
| p=1 | 0 | 3000 | 0 |
| p=2 | 300 | 4999 | 0 |

| Q_{pj} | j=1 | j=2 | j=3 |
|----------|-----|------|-----|
| p=1 | 0 | 0 | 0 |
| p=2 | 599 | 1199 | 0 |

P=1

| Z_{plj} | j=1 | j=2 | j=3 |
|-----------|------|------|-----|
| L=1 | 3000 | 0 | 0 |
| L=2 | 0 | 0 | 0 |
| L=3 | 0 | 4000 | 0 |

P=2

| Z_{plj} | j=1 | j=2 | j=3 |
|-----------|-----|-----|-----|
| L=1 | 0 | 0 | 0 |
| L=2 | 0 | 0 | 0 |
| L=3 | 0 | 0 | 0 |

P=1

| X_{pjn} | n=1 | n=2 | n=3 |
|-----------|------|------|-----|
| j=1 | 3000 | 0 | 0 |
| j=2 | 0 | 0 | 0 |
| j=3 | 0 | 4000 | 0 |

P=2

| X_{pjn} | n=1 | n=2 | n=3 |
|-----------|-----|-----|-----|
| j=1 | 0 | 0 | 0 |
| j=2 | 0 | 0 | 0 |
| j=3 | 0 | 0 | 0 |

| S_{lj} | j=1 | j=2 | j=3 |
|----------|-----|-----|-----|
| L=1 | 1 | 0 | 0 |
| L=2 | 1 | 0 | 0 |
| L=3 | 0 | 1 | 0 |

| F_{jn} | n=1 | n=2 | n=3 |
|----------|-----|-----|-----|
| j=1 | 1 | 0 | 0 |
| j=2 | 1 | 0 | 0 |
| j=3 | 0 | 1 | 0 |

Conclusions


This study explains the overall structure of the supply chain in phase space that considered different costs in real world in supply chain. The structure of objective function has been considered to maximize the difference between total revenue and available different expenditure in the real world. The output of model is the optimal volume of sending and receiving Shipment from suppliers to distribution centers (liaison between suppliers and retailers) as well as the optimal size of the sending and receiving of Shipment from distribution centers to the retailers that used from the queuing model M/G/1. In the successive researches, the multiple criteria decision making method can be used in order to the distribution centers will be selected according to various criteria. To apply and analyze this issue can have a rightful effects on being economical and commercial of distribution systems especially in the food industry area that leads to proper management of market, to reduce the system costs, to increase the customers' satisfaction, to prevent damage, corruption and waste of food and performance and to increase the efficiency effectiveness of the distribution systems in general level.

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